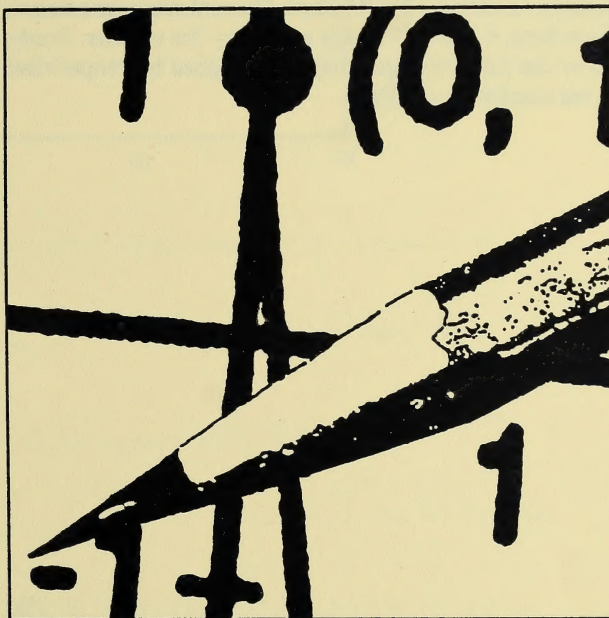




MATHEMATICS 3

LEARNING FACILITATOR'S MANUAL



UNIT 5: GEOMETRIC VECTORS AND THEIR APPLICATION



Distance
Learning

Alberta
EDUCATION

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Unit 5
Geometric Vectors and Their Application
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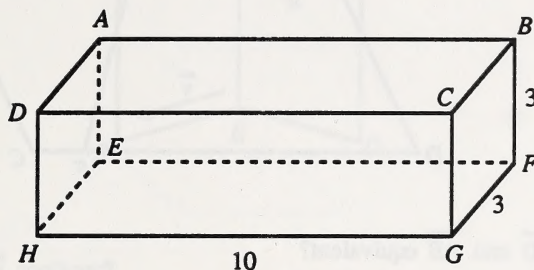
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Topic 1: Introduction to Three-Dimensional Vectors

⑥

1. In the following diagram of a rectangular prism, find the measure of $\angle CEG$.



$$EG^2 = 10^2 + 3^2$$

$$= 109$$

$$EG = \sqrt{109}$$

$$\doteq 10.44$$

$$\tan \angle CEG = \frac{3}{10.44}$$

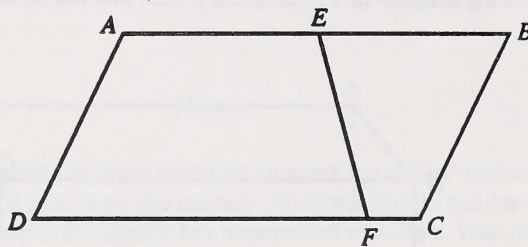
$$= 0.2873$$

$$\therefore \angle CEG \doteq 16^\circ$$

The angle is approximately 16° .

5

2. $ABCD$ is a parallelogram as shown.



- a. Are $\overrightarrow{CD}$ and $\overrightarrow{AB}$ equivalent?

No, because the vectors have different directions.

- b. Are $\overrightarrow{BE}$ and $\overrightarrow{CF}$ collinear?

Yes, the vectors are parallel.

- c. Are $\overrightarrow{AE}$ and $\overrightarrow{FC}$ coplanar?

Yes, because the vectors are in the same plane.

- d. Are $|\overrightarrow{AC}|$ and $|\overrightarrow{DB}|$ equal?

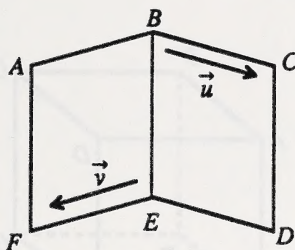
No, because they have different magnitudes.

- e. Are $\overrightarrow{BE}$ and $\overrightarrow{FD}$ equivalent vectors?

No, because they have the same direction but different magnitudes.

4

3. In the following diagram $ABEF$ and $BCDE$ are parallelograms.



- a. Are $\vec{u}$ and $\vec{v}$ coplanar?

Yes, any two vectors are coplanar.

- b. Are $\overrightarrow{AF}$ and $\overrightarrow{CD}$ equivalent?

Yes, the vectors have the same magnitude and direction.

- c. Does $|\overrightarrow{FA}|$ equal $|\overrightarrow{CD}|$?

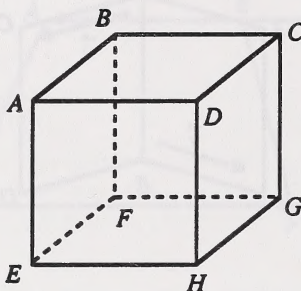
Yes, the magnitudes of the vectors are equal.

- d. Are $\vec{v}$ and $\vec{u}$ collinear?

No, they have different directions.

5

4. Examine the following cube.



a. Are all the vectors equivalent?

No, because of the definition of equivalent vectors.

b. Are $\overrightarrow{BC}$ and $\overrightarrow{DH}$ coplanar?

Yes, because any two vectors are coplanar.

c. Are $\overrightarrow{BC}$ and $\overrightarrow{HE}$ collinear?Yes, because $\overrightarrow{BC}$ and $\overrightarrow{HE}$ are parallel vectors.d. Are $|\overrightarrow{AE}|$ and $|\overrightarrow{BC}|$ equivalent?

Yes, because the measures of all the sides are equal.

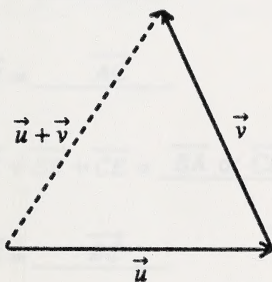
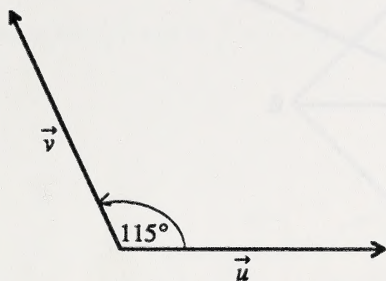
e. Is $|\overrightarrow{AH}|$ greater than $|\overrightarrow{BG}|$?

No, they are equal because they are the diagonals of sides of the same cube.

5. Use a compass and straightedge to find the resultant of the following vector additions.

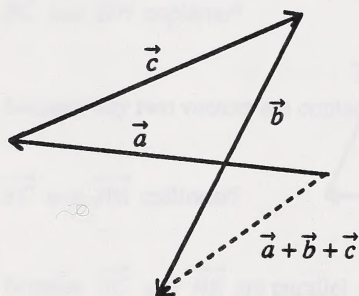
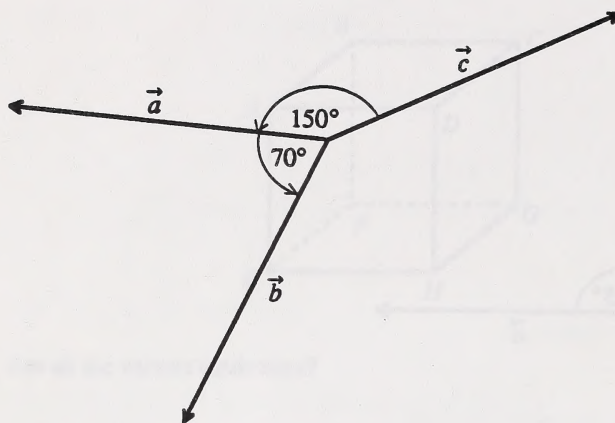
②

a. $\vec{u} + \vec{v}$



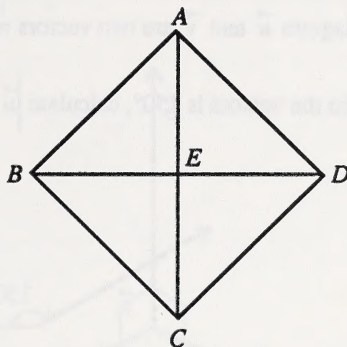
③

b. $\vec{a} + \vec{b} + \vec{c}$



5

6. For rhombus $ABCD$ state a single vector which is equivalent to the following.



- a. $\overrightarrow{AD} + \overrightarrow{EA} = \underline{\overrightarrow{BE} \text{ or } \overrightarrow{ED}}$
- b. $\overrightarrow{AD} + \overrightarrow{AB} = \underline{\overrightarrow{AC}}$
- c. $\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{EB} + \overrightarrow{CE} = \underline{\overrightarrow{BA} \text{ or } \overrightarrow{CD}}$
- d. $\overrightarrow{BE} - \overrightarrow{CE} = \underline{\overrightarrow{BC}}$
- e. $\overrightarrow{EC} - \overrightarrow{AB} + \overrightarrow{DA} = \underline{\overrightarrow{EA}}$

Topic 1

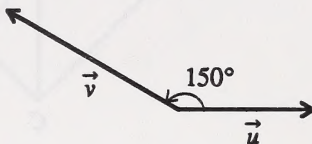
_____ marks

Topic 2: Vector Properties

④

1. In the following diagram $\vec{u}$ and $\vec{v}$ are two vectors where $|\vec{u}| = 4$ units and $|\vec{v}| = 6$ units of length.

If the angle between the vectors is 150° , calculate $|\vec{u} + \vec{v}|$ to two decimal places.



$$|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos 30^\circ$$

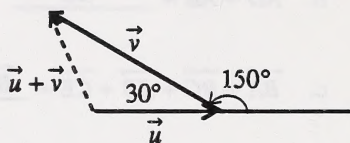
$$= 4^2 + 6^2 - 2(4)(6)\cos 30^\circ$$

$$= 16 + 36 - 48\left(\frac{\sqrt{3}}{2}\right)$$

$$= 10.43$$

$$|\vec{u} + \vec{v}| = \sqrt{10.43}$$

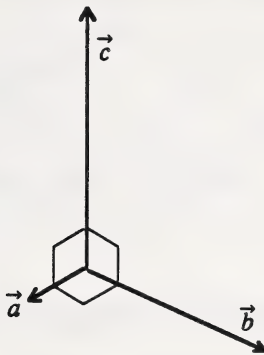
$$\doteq 3.23$$



4

2. Three perpendicular forces act on an object as shown in the following diagram.

If $|\vec{a}| = 1$ N, $|\vec{b}| = 3$ N, and $|\vec{c}| = 4$ N, find the magnitude of the resultant to one decimal place.



Let $\vec{r}$ be the resultant of $\vec{a}$ and $\vec{b}$, and let $\vec{R}$ be the resultant of $\vec{a}$, $\vec{b}$, and $\vec{c}$.

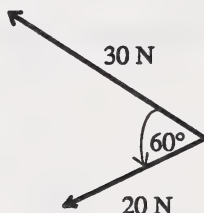
$$\begin{aligned}\vec{r} &= \vec{a} + \vec{b} \\ |\vec{r}|^2 &= 1^2 + 3^2 \\ &= 10 \\ \therefore |\vec{r}| &= \sqrt{10}\end{aligned}$$

$$\begin{aligned}\vec{R} &= \vec{r} + \vec{c} \\ |\vec{R}|^2 &= 10 + 4^2 \\ &= 26 \\ \therefore |\vec{R}| &= \sqrt{26} \\ &\approx 5.1\end{aligned}$$

The magnitude of the resultant is approximately 5.1 N.

5

3. Two forces of 20 N and 30 N act on an object. The angle between the two forces is 60° . Calculate the magnitude and direction of the equilibrant force. Express the direction with respect to the 30 N force as shown in the following diagram.



Let $\vec{R}$ be the resultant of the two forces and $\vec{E}$ be the equilibrant of the two forces.

$$\begin{aligned} |\vec{R}|^2 &= 30^2 + 20^2 - (2)(30)(20 \cos 120^\circ) \\ &= 900 + 400 + 600 \\ &= 1900 \end{aligned}$$

$$\begin{aligned} \therefore |\vec{E}| = |\vec{R}| &= \sqrt{1900} \\ &\doteq 43.6 \end{aligned}$$

$$\frac{20}{\sin \theta} = \frac{43.59}{\sin 120^\circ}$$

$$\sin \theta = \frac{20 \left(\frac{\sqrt{3}}{2} \right)}{43.59}$$

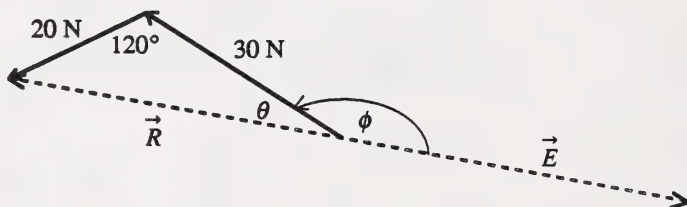
$$= \frac{17.32}{43.59}$$

$$= 0.3973$$

$$\theta \doteq 23.4^\circ$$

$$\therefore \phi = 180^\circ - \theta$$

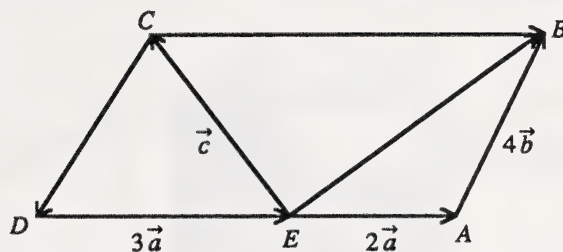
$$\doteq 156.6^\circ$$



The equilibrant is approximately 43.6 N and makes a 156.6° angle with the 30 N force.

⑥

4. In the following diagram $\overrightarrow{DE} = 3\vec{a}$, $\overrightarrow{EA} = 2\vec{a}$, $\overrightarrow{AB} = 4\vec{b}$, and $\overrightarrow{EC} = \vec{c}$.



Express $\overrightarrow{CB}$ and $\overrightarrow{CD}$ in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.

$$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$$

$$= 2\vec{a} + 4\vec{b}$$

$$\overrightarrow{CB} = \overrightarrow{CE} + \overrightarrow{EB}$$

$$= -\vec{c} + 2\vec{a} + 4\vec{b}$$

$$= 2\vec{a} + 4\vec{b} - \vec{c}$$

$$\overrightarrow{CD} = \overrightarrow{CE} + \overrightarrow{ED}$$

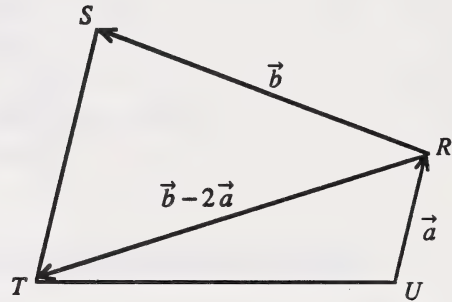
$$= -\vec{c} - 3\vec{a}$$

6

5. $RSTU$ is a quadrilateral with $\overrightarrow{RS} = \vec{b}$, $\overrightarrow{UR} = \vec{a}$, and $\overrightarrow{RT} = \vec{b} - 2\vec{a}$. Express $\overrightarrow{ST}$ and $\overrightarrow{UT}$ in terms of $\vec{a}$ and $\vec{b}$.

$$\begin{aligned}\overrightarrow{ST} &= \overrightarrow{SR} + \overrightarrow{RT} \\ &= -\vec{b} + (\vec{b} - 2\vec{a}) \\ &= -2\vec{a}\end{aligned}$$

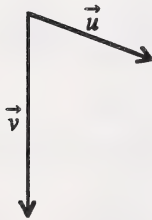
$$\begin{aligned}\overrightarrow{UT} &= \overrightarrow{UR} + \overrightarrow{RT} \\ &= \vec{a} + (\vec{b} - 2\vec{a}) \\ &= \vec{b} - \vec{a}\end{aligned}$$



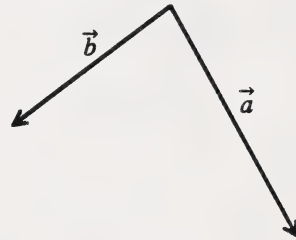
5

6. Using a compass and straightedge, construct the resultant of the following vector subtractions.

a. $\vec{v} - \vec{u}$

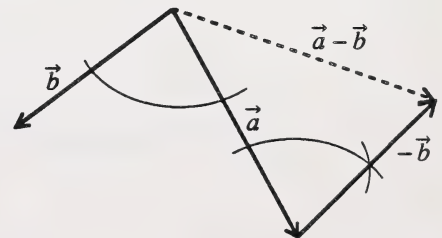
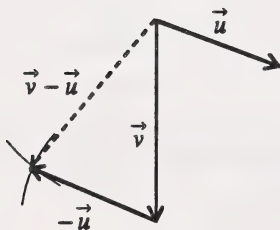


b. $\vec{a} - \vec{b}$



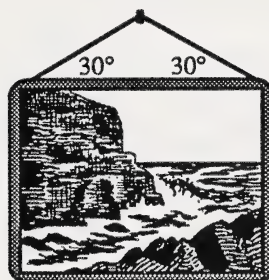
Topic 2

_____ marks



Topic 3: Vector Applications

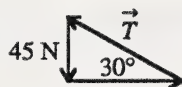
- 5 1. A 90 N picture is hung by cords as shown. What is the tension (force) in each of the cords?



Let $\vec{T}$ be the tension in each cord.

$$\left(\frac{1}{2}\right)(90) = 45$$

$$\begin{aligned} |\vec{T}| &= \frac{45}{\sin 30^\circ} \\ &= (45)(2) \\ &= 90 \end{aligned}$$



The tension in each cord is 90 N.

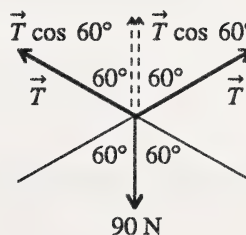
Alternate Method:

For equilibrium the upward vertical forces must equal the downward vertical forces. The vertical component of each tension force is $|\vec{T}| \cos 60^\circ$.

$$|\vec{T}| \cos 60^\circ + |\vec{T}| \cos 60^\circ = 90$$

$$2|\vec{T}| \cos 60^\circ = 90$$

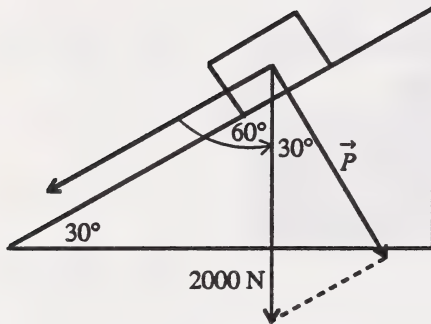
$$\begin{aligned} |\vec{T}| &= \frac{90}{2 \cos 60^\circ} \\ &= 90 \end{aligned}$$



Therefore, the tension in each cord is 90 N.

④

2. An object weighing 2000 N is being pushed along a smooth ramp which forms an angle of 30° to the horizontal. Find the force on the ramp. (This is the force in the direction perpendicular to the ramp.)



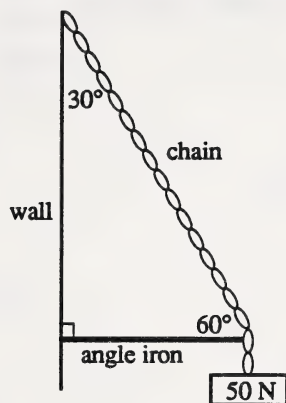
Let $\vec{P}$ be the force exerted by the object on the ramp.

$$\begin{aligned} |\vec{P}| &= 2000 \cos 30^\circ \\ &= (2000)(0.8660) \\ &\doteq 1732 \end{aligned}$$

The force exerted down the ramp is approximately 1732 N.

5

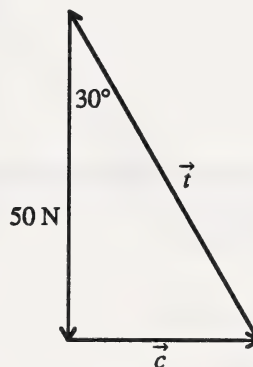
3. A 50 N sign hangs from the end of a piece of angle iron as shown. A chain is used to connect the end of the angle iron and the sign. Find the tension in the chain and the compression of the angle iron.



Let $\vec{t}$ be the tension in the chain and $\vec{c}$ be the compression of the angle iron.

$$\begin{aligned}\cos 30^\circ &= \frac{50}{|\vec{t}|} \\ |\vec{t}| &= \frac{50}{\cos 30^\circ} \\ &= \frac{(50)(2)}{\sqrt{3}} \\ &\doteq 58\end{aligned}$$

$$\begin{aligned}\tan 30^\circ &= \frac{|\vec{c}|}{50} \\ |\vec{c}| &= 50 \tan 30^\circ \\ &= (50)(0.5773) \\ &\doteq 29\end{aligned}$$



Therefore, the tension is approximately 58 N and the compression is approximately 29 N.

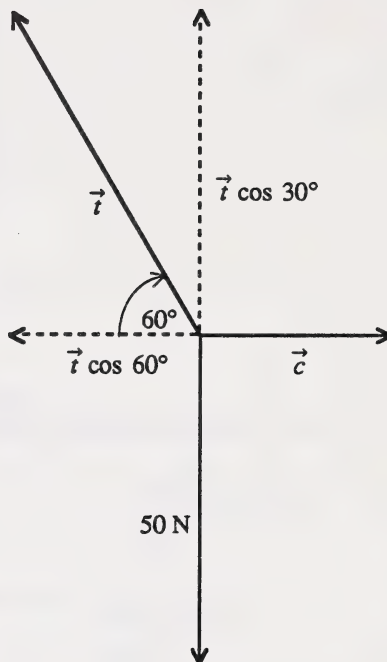
Alternate Method:

Resolve the tension force into its rectangular components. For equilibrium the upward vertical force must equal the downward vertical force.

$$\therefore |\vec{t}| \cos 30^\circ = 50$$

$$|\vec{t}| = \frac{50}{\cos 30^\circ}$$

$$\doteq 58^\circ$$



For equilibrium the horizontal force to the right must equal the horizontal force to the left.

$$|\vec{c}| = |\vec{t}| \cos 60^\circ$$

$$|\vec{c}| = 58(0.5)$$

$$\doteq 29$$

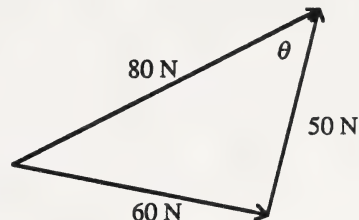
Therefore, the tension is approximately 58 N and the compression is approximately 29 N.

5

4. Draw a vector to represent an 80 N force. If this force is resolved into two non-rectangular components of 50 N and 60 N, find the angle between the 80 N force and the 50 N force.

$$\begin{aligned}
 60^2 &= 50^2 + 80^2 - (2)(50)(80 \cos \theta) \\
 3600 &= 2500 + 6400 - 8000 \cos \theta \\
 8000 \cos \theta &= 5300 \\
 \cos \theta &= \frac{53}{80} \\
 &= 0.6625 \\
 \therefore \theta &\doteq 48.5^\circ
 \end{aligned}$$

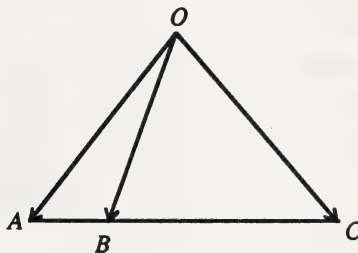
The angle is approximately 48.5° .



5

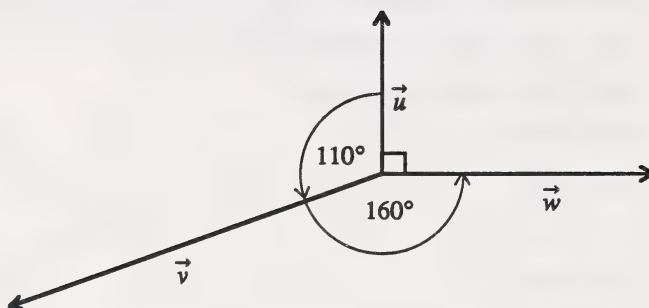
5. A , B , and C are collinear points. $BC = 3AB$ and O is any point not on segment AC . Prove that

$$3\overrightarrow{OA} + \overrightarrow{OC} = 4\overrightarrow{OB}.$$



$$\begin{aligned}
 \overrightarrow{OC} &= \overrightarrow{OB} + \overrightarrow{BC} \\
 \overrightarrow{OA} &= \overrightarrow{OB} + \overrightarrow{BA} \\
 &= \overrightarrow{OB} - \overrightarrow{AB} \\
 \therefore 3\overrightarrow{OA} + \overrightarrow{OC} &= 3(\overrightarrow{OB} - \overrightarrow{AB}) + (\overrightarrow{OB} + \overrightarrow{BC}) \\
 &= 3\overrightarrow{OB} - 3\overrightarrow{AB} + \overrightarrow{OB} + \overrightarrow{BC} \\
 &= 4\overrightarrow{OB} - 3\overrightarrow{AB} + \overrightarrow{AB} \\
 &= 4\overrightarrow{OB}
 \end{aligned}$$

7. 6. Three coplanar vectors are shown in the following diagram.

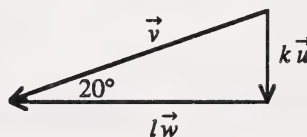


If $|\vec{u}| = 5$, $|\vec{v}| = 12$, and $|\vec{w}| = 8$, express $\vec{v}$ as a linear combination of $\vec{u}$ and $\vec{w}$.

Let $\vec{v} = k\vec{u} + l\vec{w}$.

$$\begin{aligned} |k\vec{u}| &= |\vec{v}| \sin 20^\circ \\ 5|k| &= 12 \sin 20^\circ \\ &= 12(0.34202) \\ &= 4.1042 \\ \therefore |k| &= 0.8208 \end{aligned}$$

$$\begin{aligned} |l\vec{w}| &= |\vec{v}| \cos 20^\circ \\ 8|l| &= 12 \cos 20^\circ \\ |l| &= \frac{11.2763}{8} \\ |l| &= 1.4095 \end{aligned}$$



According to the diagram $k, l < 0$ since $k\vec{u}$ is in the opposite direction to $\vec{u}$, and $l\vec{w}$ is in the opposite direction to $\vec{w}$. (This means that k and l are negative.)

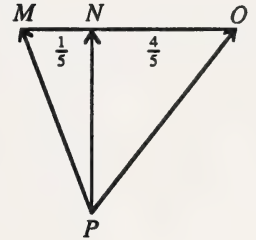
Therefore, $\vec{v} = -0.8208\vec{u} - 1.4095\vec{w}$.

- ④ 7. M, N , and O are collinear points where $MN:NO = 1:4$. P is any point not on the segment MO . Express $\overrightarrow{PN}$ as a linear combination of $\overrightarrow{PM}$ and $\overrightarrow{PO}$.

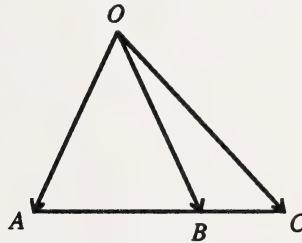
Let segment $MN = 1$ and segment $NO = 4$; thus, segment $MO = 5$.

Thus, $\frac{MN}{NO} = \frac{1}{4}$ and $\frac{NO}{MO} = \frac{4}{5}$.

Therefore, $\overrightarrow{PN} = \frac{4}{5}\overrightarrow{PM} + \frac{1}{5}\overrightarrow{PO}$.



- ⑤ 8. If $\overrightarrow{OA} = 3\overrightarrow{OB} - 2\overrightarrow{OC}$, prove that A, B , and C are collinear and determine the ratio $AB:BC$.

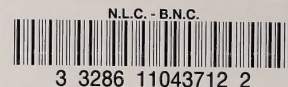


$$\begin{aligned}
 \overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\
 &= \overrightarrow{OB} - \overrightarrow{OA} \\
 &= \overrightarrow{OB} - 3\overrightarrow{OB} + 2\overrightarrow{OC} \\
 &= 2\overrightarrow{OC} - 2\overrightarrow{OB} \\
 &= 2(\overrightarrow{OB} + \overrightarrow{BC}) - 2\overrightarrow{OB} \\
 &= 2\overrightarrow{OB} + 2\overrightarrow{BC} - 2\overrightarrow{OB} \\
 &= 2\overrightarrow{BC}
 \end{aligned}$$

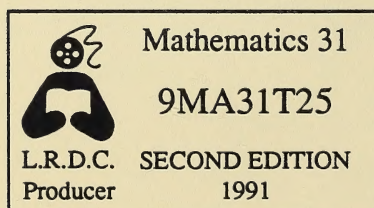
Topic 3

$\overrightarrow{AB}$ is a scalar multiple of $\overrightarrow{BC}$. Therefore, A, B , and C are collinear and $AB:BC = 2:1$.

_____ marks



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Mathematics 31

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